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Category Theory in Geometrical Product Specifications and Verification (GPS)

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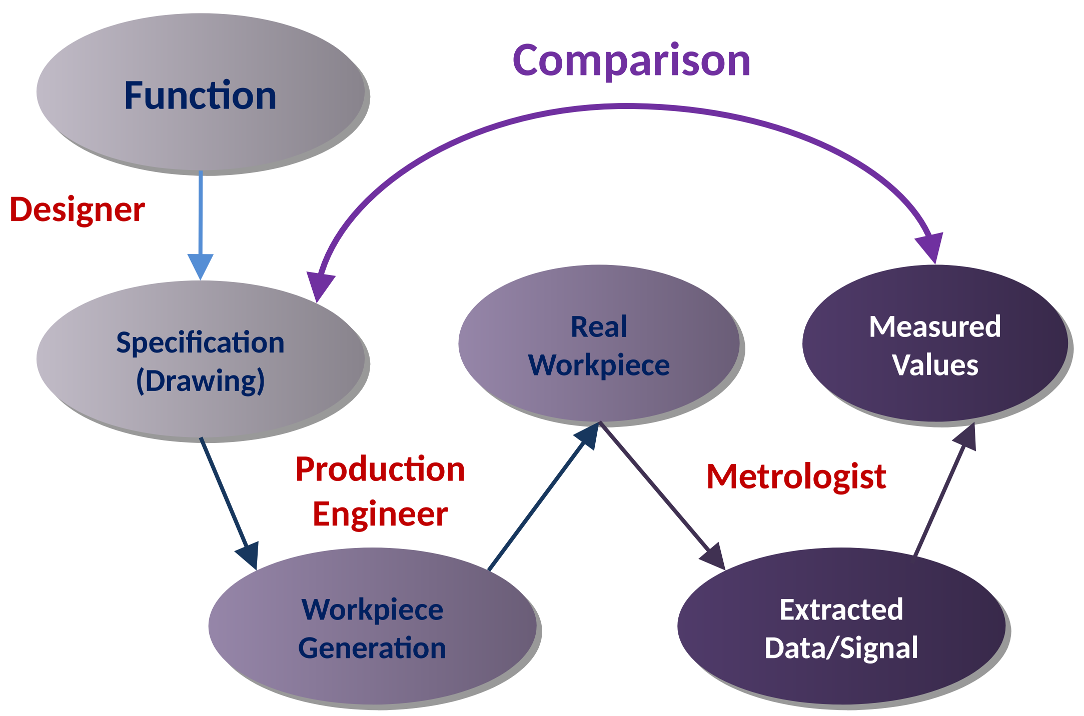
What is GPS ?

GPS - A Synergy between disciplines a critical factor

GPS operates on Geometrical Products form:

Design -> Manufacturing -> Metrology -> Product Life -> End of Life



GPS provides a unified system:

- A common Language
- A common Usage
- A common understanding

Amongst Designers, Production Engineers and Metrologists.

ISO GPS Standards provide a basis for legal contracts amongst manufacturers, their supply chain, product lifecycle managers and customers.

Category Model of Stable Measurement

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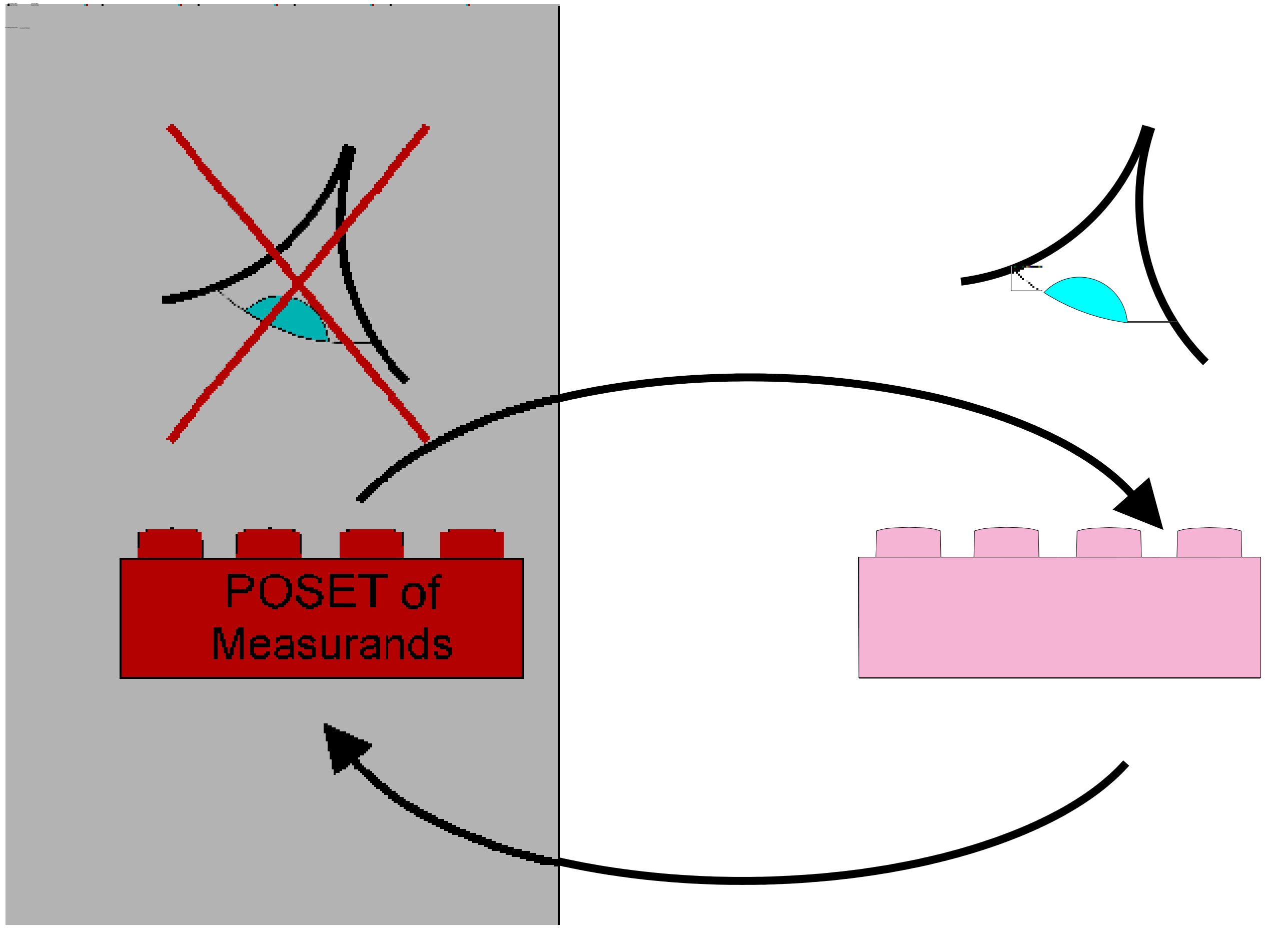
A measuring procedure is considered stable if a small difference in the Observed measured value implies a small difference in the measurand.

In Topology a neighbourhood (open set) can be used to define a small difference and the above definition implies that the mapping is a continuous mapping

There is a one-to-one duality between discrete topologies and partial pre-order categories

Category theory provides a philosophical framework and a toolbox of techniques for the investigation of inverse measurement models.



Duality Principle

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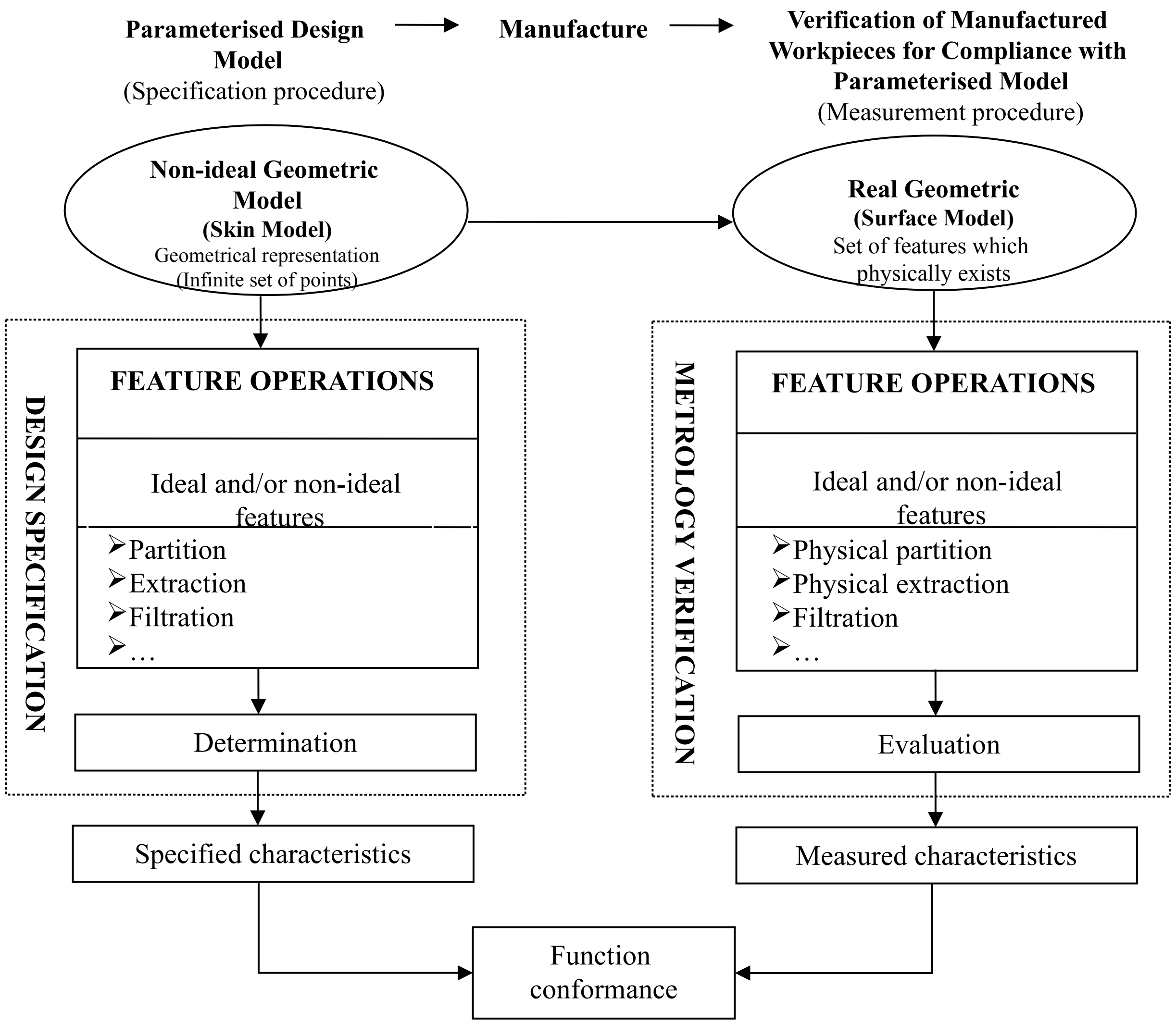
- Duality principle (ISO 8015:2011)

  "a specification operator is independent of any measurement characteristics; and the specification operator is physically realised in a verification operator, which is intended to mirror the specification operator, but independent of such."

- Translation:
  - a set of specification operation will be mapped full faithfully to a set of verification operation.
  - Specification ⇒ Verification

Category Concept – Adjoint Functors

Specification ⟺ Verification



The GPS Matrix (ISO 14638)

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The GPS Matrix was an early attempt to organise the standards documents within the GPS system.

Each ISO document is allocated to a cell (or a group of contiguous cells).

This indicates possible missing ISO documents or possible duplicate ISO documents.

	Chain links						
	A	B	C	D	E	F	G
	Symbols and indications	Feature requirements	Feature properties	Conformance and non-conformance	Measurement	Measurement equipment	Calibration
Size							
Distance							
Form							
Orientation							
Location							
Run-out							
Profile surface texture			•			•	
Areal surface texture							

Chain of GPS Standards

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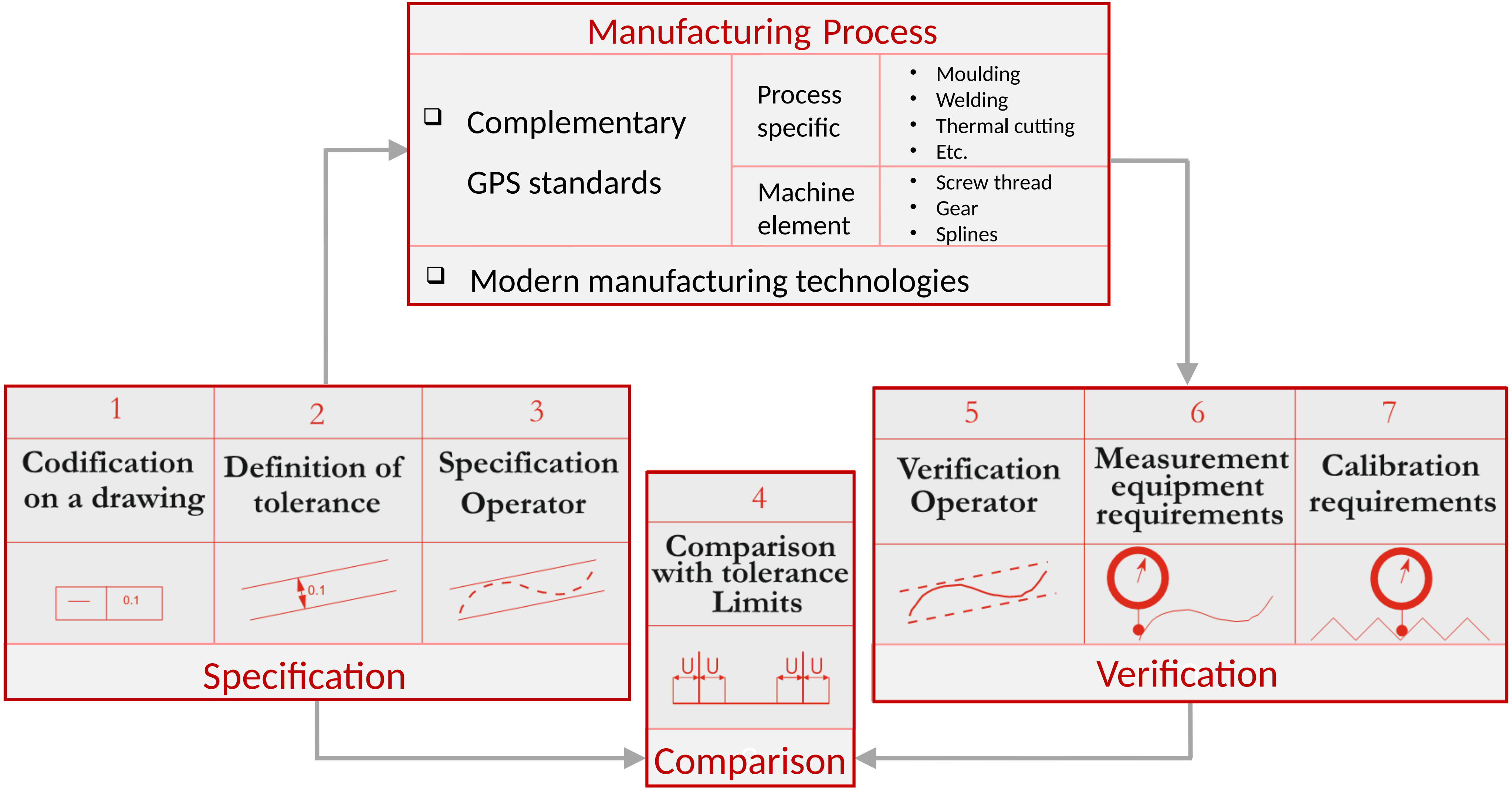

Flow of information between ISO documents

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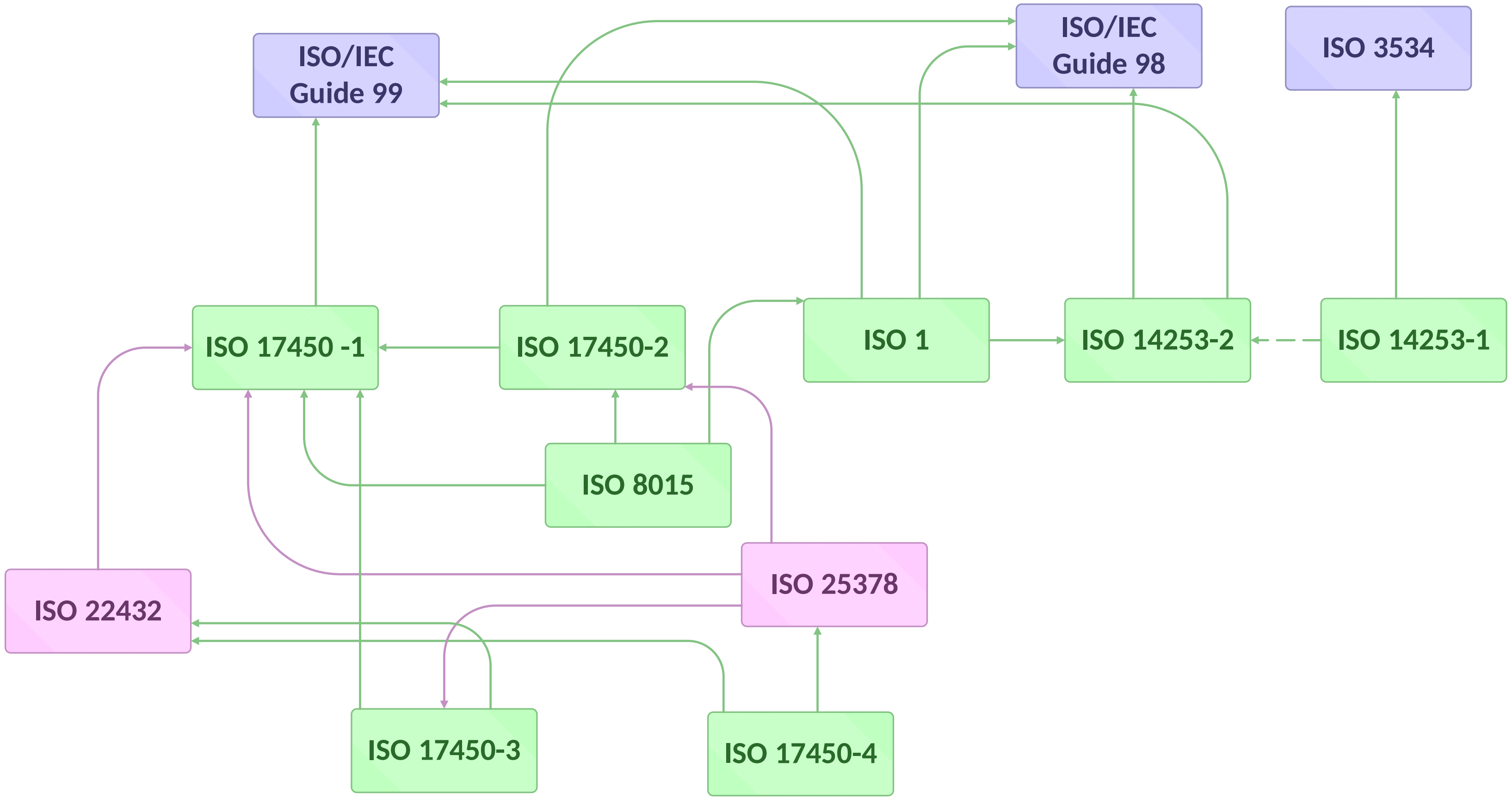
It is proposed that the current 150 ISO documents be partitioned into four groups that form a hierarchy:

1. Foundational;
2. Fundamental;
3. Matrix documents;
4. Other General documents;

As can be seen:

- They do not start with a single document

- Some foundational documents depend on fundamental documents.
- The information flow is a mess!



Current ISO GPS structure for Foundational documents (green)

Flow of information between ISO documents

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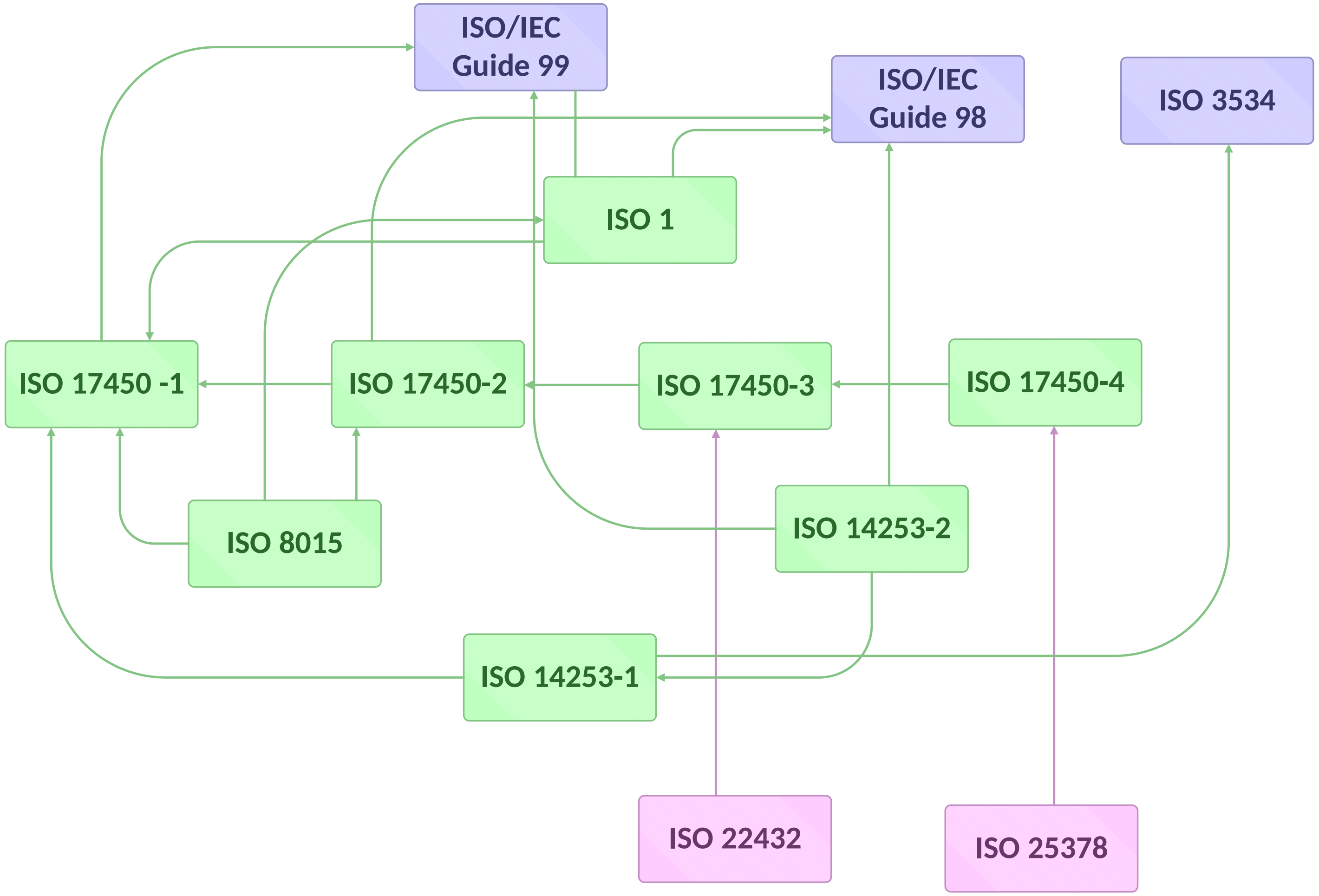
As can be seen:

- It starts with a single document ISO 17450-1
- Foundational documents do not depend on fundamental documents.
- The information flow is similar to a deductive system and takes the form of a POSET
- The structure can be captured in the newly developed Category Semantic Language (CSL)



Proposed ISO GPS structure for Foundational documents (green)

AI Ready Smart Knowledge

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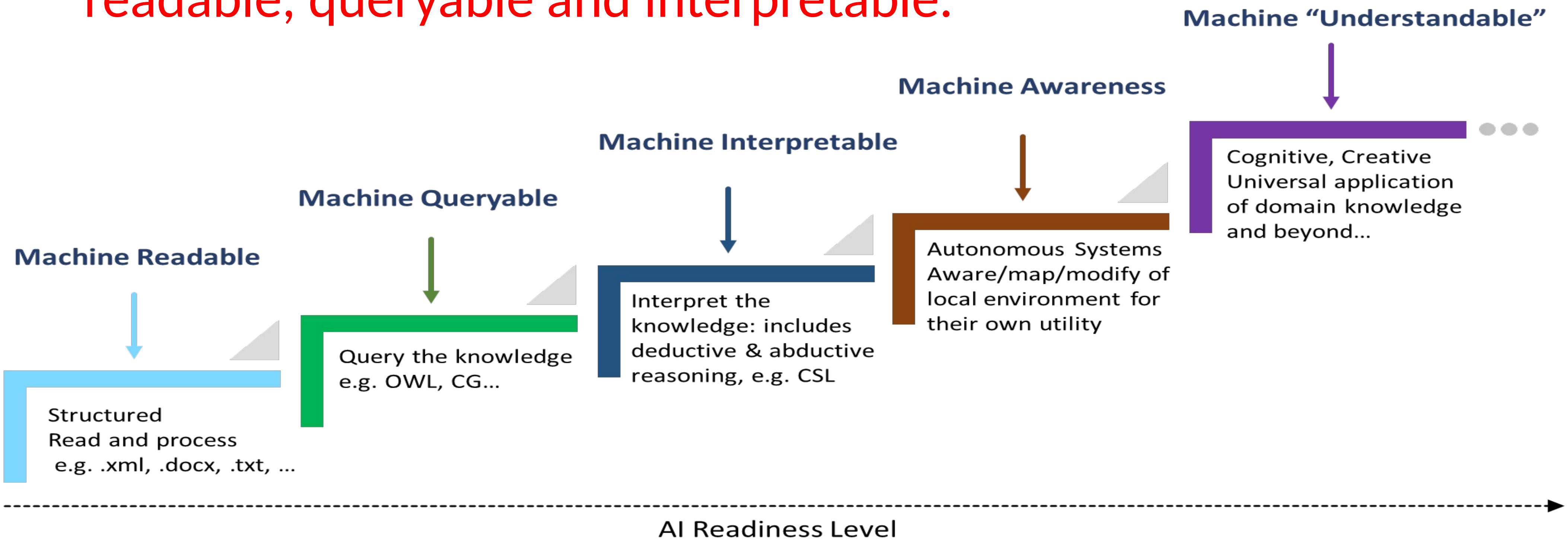
Thus it is essential that human knowledge is translated into AI ready smart knowledge for future AI systems to use, which are:

readable, queryable and interpretable.



Smart Manufacturing

- *Informatics GPS System*

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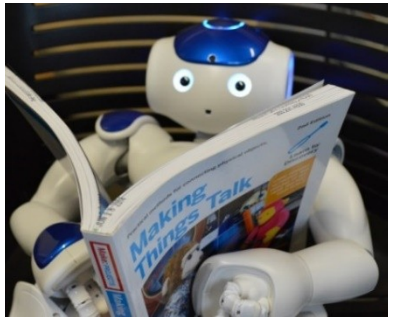


Informatics ISO GPS System

- Standards
- Supporting knowledge
- Related manufacturing materials
- etc.

Knowledge retrieval & reasoning

Smart master controller & interface

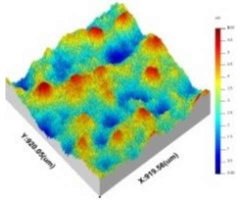
- Direct access to Library knowledge and measurement analytic results
- Interactive knowledge interface for end-users

Production tasks

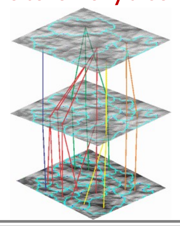
Measurement data analytics & Manufacturing control

Measurement analytic results

Verification

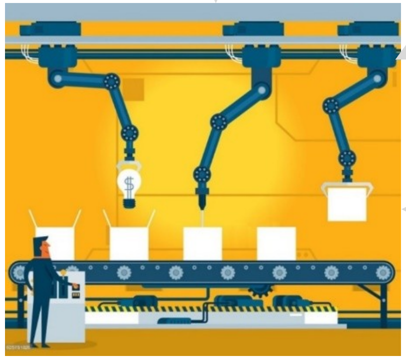


Data analytics



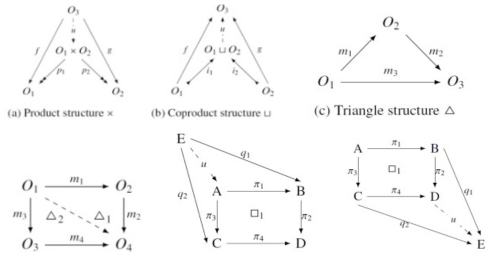
Measurement data flow and control

Production line with embedded metrology and control



Category Semantic Language

CSL Syntax



CSL Reasoning

$\prod$ Rule

For $S_{\square_1}(S_{\triangle_1}, S_{\triangle_2})$ if $\nexists\, O(O_t) \in \mathcal{C}$, or
For $\exists \pi_1$ and π_2, $dom(\pi_1) = dom(\pi_2)$ if $\exists O(O_t) \in \mathcal{C}$
then $\exists \lambda_1 : B \rightarrow O(O_t), \lambda_2 : C \rightarrow O(O_t)$;
$S_{\square_1}.S_{\triangle_1}.M_1 = \pi_2$; $S_{\square_1}.S_{\triangle_2}.M_1 = \pi_1$; $S_{\square_1}.S_{\triangle_1}.M_2 = \lambda_1$; $S_{\square_1}.S_{\triangle_2}.M_2 = \lambda_2$,
if $\lambda_1(M_m \mid M_i) \& \lambda_2(M_m \mid M_i)$,
then $\exists S_{\prod}(S_{\square_1}, \{S_{\square'}\}, \{u\})$,
$\forall S_{\square'}$, $\exists! cod(S_{\square'}, S_{\triangle_1}, M_2)$;
$\{u\} = Hom(dom(S_{\square'}.S_{\triangle_1}.M_1), dom(\pi_2))$;
if $\exists S_{\times} \in \mathcal{C}$,
then
$\exists\, dom(\lambda_1) \times dom(\lambda_2)$;
$\exists\, p_1.M_p : dom(\lambda_1) \times dom(\lambda_2) \rightarrow dom(\lambda_2)$;
$\exists\, p_2.M_p : dom(\lambda_1) \times dom(\lambda_2) \rightarrow dom(\lambda_1)$;
$\exists!\, u.M_u : dom(\lambda_1) \times dom(\lambda_2) \rightarrow dom(\pi_1)$.

CSL Algorithms

procedure STEP 1: SEARCH IN MORPHISM SET M_i
  for $i = 0, i < t, i++$ **do**
    if $(i+1) \leq x \leq (t-1)$;
$dom(M_i) == dom(M_x) \& NoMorphism(cod(M_i), cod(M_x))$
$\& M_i(M_r)$; or
$dom(M_i) == cod(M_x) \mid cod(M_i) == dom(M_x)$
$\& NoMorphism(cod(M_x), dom(M_i))$; or
$cod(M_i) == cod(M_x) \& NoMorphism(dom(M_x), dom(M_i))$
$\& M_i(M_x)$ **then**
      Apply "∘ Rule"
      if return a new morphism M_y, t++ **then**
        Save a new $S_{\triangle_x}$ to $\{S_{\triangle_h}\}$, and n++;
    else go to the next step

procedure STEP 2: SEARCH IN TRIANGLE STRUCTURES $\triangle_h$
  for $h = 0, h < n, h++$ **do**
    if $sameMorphism(M_3 \in S_{\triangle_h}, M_3 \in S_{\triangle_x}, (h+1) \leq x \leq (n-1)$ **then**
      Save a new rectangle structure $S_{\square_{x1}}(S_{\triangle_h}, S_{\triangle_x})$, m++;
      if $S_{\triangle_h}.M_2(M_m \mid M_i) \& S_{\triangle_x}.M_2(M_m \mid M_i)$ **then**
        Apply "∏ Rule"
        if successful **then**
          Save a new object O_x to $\{O_j\}$, a set of new morphisms $\{M_x\}$ to $\{M_i\}$, a set of new triangle structures $\{S_{\triangle_x}\}$ to $\{S_{\triangle_h}\}$, a new product structure $S_{\times_x}$ to $\{S_{\times_p}\}$, a set of new rectangle structures $\{S_{\square_x}\}$ to $\{S_{\square_k}\}$, and a new $S_{\prod_x}(S_{\square_{x1}}, \{S_{\square_x}\}, \{u\})$ to $S_{\prod_e}$.
      else if $S_{\triangle_h}.M_1(M_e \mid M_i) \& S_{\triangle_x}.M_1(M_e \mid M_i)$ **then**
        Apply "∐ Rule"
        if successful **then**

Any Questions?